

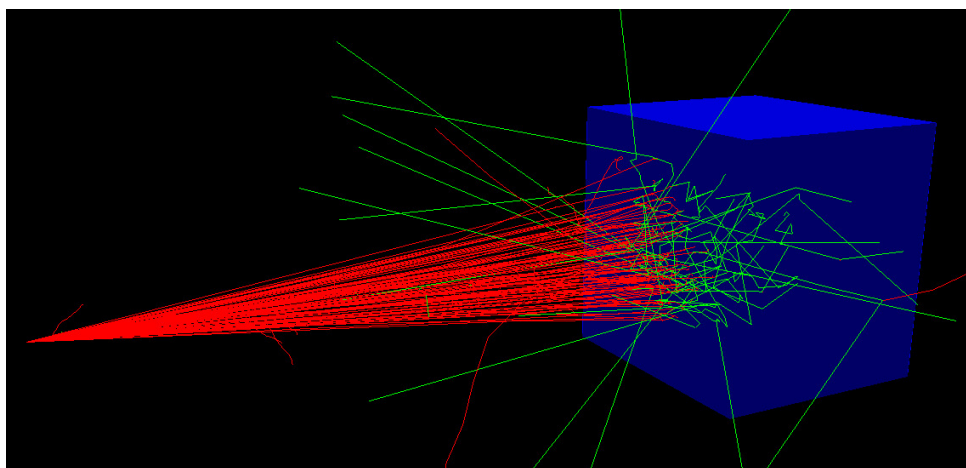
**Instituto Superior de Engenharia de Lisboa
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Departamento de Física**

Licenciatura em Engenharia Física Aplicada

**Trabalho final de Curso
Bachelor's Thesis**

Monte Carlo Simulation of Electron Beams with Typical Radiotherapy Energies

**Simulação Monte Carlo de Feixes de Elétrons
com Energias Típicas de Radioterapia**



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“Porque o SENHOR dá a sabedoria; da sua boca vem o conhecimento e o entendimento.”
Provérbios 2:6

Em primeiro lugar, gostaria de agradecer a Deus por me abençoar com esta oportunidade, concedendo-me saúde, esperança e todas as pessoas maravilhosas que integraram esta jornada.

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Paulo Rodrigues
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Abstract

Nowadays, Monte Carlo (MC) simulations are widely used in the clinical radiotherapy scenario, playing a major role in patient treatment planning. Several computational tools are available for performing these simulations, but using them correctly requires a thorough understanding of their mechanisms and foundational knowledge of radiation-matter interaction phenomena.

One prominent example is the *Geant4 Application for Tomographic Emission* (GATE), introduced by S. Jan et al. in 2004 [6]. GATE provides a series of protocols and commands built on top of the GEANT4 toolkit which are then implemented in a macro file commonly referred to as the input file.

Numerous preliminary studies were carried out to determine the simulation parameters that now make up the final input file. These parameters include the size of the simulation's world—that is, the outer volume enclosing all the other objects, within which all particle interactions algorithms take place—the phantom dimensions, the size and resolution of the actors (objects used to record values of interest), which Physics List to use (the library containing all particle-matter interaction protocols), source definition and configuration, source-to-surface distance (SSD), and many other variables that could only be understood after many hours of study and dedicated work with the tool.

The simulation model developed in this work produced percent-depth-dose (PDD) curves with uncertainties ranging from 0.8% to 0.5%, while lateral dose profiles presented uncertainties ranging from 9% to $\sim 2\%$, showing agreement of $\leq 4.4\%$ with published reference values—quantitatively demonstrating concordance with expected results. The energy spectra obtained confirmed the validity of the input file by matching the observed energies in each section of the phantom to the known physics of radiation-matter interaction. Thus, one can assert that the results represent a validation of the established model, indicating that it can be used for future simulations of electron beams in water, polystyrene or Lucite phantoms, commonly used in clinic dosimetry for quality assurance.

The importance of this work lies in the experience gained with the GATE platform and with the MC particle transport algorithm, which itself holds great academic value and, above all, career relevance in medical physics with a focus on radiotherapy.

Contents

I	Introduction	4
1.1	Thesis motivation	4
1.2	Electron beams in radiotherapy	4
1.3	Dosimetry concepts	5
1.3.1	Percent-depth-dose	5
1.3.2	Electron range	5
1.3.3	Electron beam energy specification	6
1.3.4	Build-up region	7
1.3.5	Dose profiles	8
1.4	Electron interactions	8
1.4.1	Elastic scattering	8
1.4.2	Ionization and excitation	9
1.4.3	Radiative energy loss	9
1.5	Photon interactions	9
1.5.1	Photoelectric effect	10
1.5.2	Compton scattering	10
1.5.3	Pair production	11
1.6	Why use Monte Carlo?	11
II	Materials and methods	13
2.1	GEANT4 & GATE	13
2.2	Software and hardware specifications	13
2.3	Physics list	14
2.4	Simulation setup	14
2.4.1	World	14
2.4.2	Phantom	14
2.4.3	Production cuts	15
2.4.4	Scoring	15
2.4.5	Source	16
2.5	Dose scoring and uncertainty quantification	16
2.5.1	Dose scoring	16
2.5.2	Dose uncertainty	17
2.6	Problem statement	17
III	Results and discussion	18
3.1	Percent-depth-dose curves	18
3.2	Lateral dose profiles	20
3.3	Energy spectrum	23
3.4	Discussion	24
IV	Conclusions	25
	References	25

I Introduction

The field of radiological physics emerged in the late nineteenth century, catalyzed by a sequence of landmark discoveries. Among these were Wilhelm Röntgen’s detection of X-rays in 1895, Henri Becquerel’s observation of natural radioactivity in 1896, and Marie and Pierre Curie’s subsequent isolation of radium. What began as pure scientific curiosity rapidly found medical application: X-ray imaging entered clinical practice almost immediately and radioactive sources were soon adopted for therapeutic purposes. Consequently, the precise characterization of energy deposition by ionizing radiation in human tissue became increasingly critical for tumor treatments. Louis Harold Gray formally defined this energy deposition as the *absorbed dose* in the 1930s, thereby establishing the foundation for contemporary radiation dosimetry.

Early dose calculations relied on analytical algorithms that, although computationally efficient, frequently fell short when confronted with complex geometries and tissue inhomogeneities. To overcome these shortcomings, Monte Carlo (MC) methods—first conceived in the late 1940s to model neutron transport in nuclear weapons research—were repurposed for medical dosimetry in the 1960s. By simulating individual particle–matter interactions, MC techniques showed markedly higher accuracy in dose prediction than do purely analytical approaches. The trajectory of MC’s adoption in medical physics has paralleled advances in computing power: what originally demanded supercomputers in the 1980s became feasible on personal workstations by the 2000s, facilitated by dedicated software frameworks.

Today, MC simulations are indispensable both in radiotherapy research and increasingly in routine clinical practice. Contemporary treatment-planning systems incorporate MC-based dose engines, while investigators employ these methods across a spectrum of applications. The democratization of high-fidelity MC tools has unlocked explorations of radiation-transport phenomena that were previously intractable, and it is against this backdrop of computational innovation that the present work applies MC methods to electron-beam dosimetry.

1.1 Thesis motivation

My interest in radiation physics developed during my undergraduate degree through computational modeling projects and laboratory experiments with cobalt-60 and cesium-137 sources. This blend of computational and experimental experience naturally steered my thesis toward MC simulation in radiotherapy.

Consequently, the primary objective of this work was to familiarize with MC particle transport techniques, particularly using the *Geant4 Application for Tomographic Emission* (GATE) toolkit (Section 2.1). This involved an intensive study of the physical and geometric parameters used in GATE input files through preliminary simulations of mono-energetic beams (photons and electrons) incident on water phantoms. This preliminary study provided deeper understanding of radiation-matter interaction processes for electrons and photons with energies typical of radiotherapy.

Principal emphasis was given to simulating electron percentage-depth-doses (PDDs) across various energies. Analysis of the results led to the study of fundamental elements of clinical electron beam dosimetry such as the build-up region, range, and dose distribution characteristics (Section 1.3). This foundational knowledge of MC techniques and electron beam physics provides essential skills for clinical medical physics applications, establishing the groundwork for future research in complex beam and geometries configurations.

This thesis serves as both the capstone of my Applied Physics Engineering degree and a foundational step toward specialization in clinical medical physics.

1.2 Electron beams in radiotherapy

Clinical electron beams span a therapeutic energy range from approximately 4 to 25 MeV, carefully selected to optimize dose distribution for different treatment scenarios. The selection of electron beam energy directly determines the depth-dose characteristics and therefore the clinical applicability. Unlike photon beams, which exhibit exponential attenuation and deposit dose throughout their entire path, electron beams demonstrate a characteristic build-up region up to a maximum at a certain depth referred to as the electron beam depth of dose maximum, $Z_{D_{\max}}$. Beyond $Z_{D_{\max}}$, the dose drops off rapidly and levels off at a small low level dose component referred to as the bremsstrahlung tail. These features offer a distinct clinical advantage over the conventional X ray modalities in the treatment of superficial tumours (Fig. 1.1).

Modern linear accelerators (LINACs) generate electron beams through acceleration of electrons in radio-frequency cavities. These beams exhibit nearly monoenergetic characteristics, as they leave the accelerator. However, as they traverse, the interaction with these materials result in the loss energy. Consequently, the clinical beam reaching the patient surface has an incident mean energy $\bar{E}_0$ lower than the nominal accelerator energy.

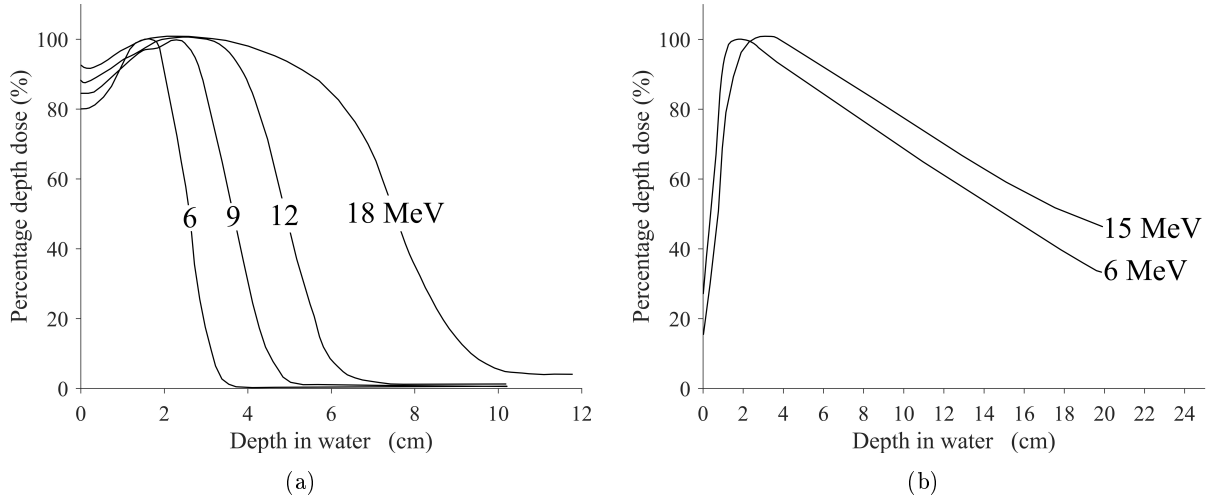


Figure 1.1: Typical central axis PDD curves in water for a $10 \times 10 \text{ cm}^2$ field size (Section 1.3.5) and an source to surface distance (SSD) of 100 cm for (a) electron beams with energies of 6, 9, 12 and 18 MeV and (b) photon beams with energies of 6 and 15 MeV [1].

1.3 Dosimetry concepts

The fundamental quantity in radiation dosimetry is the *absorbed dose*, defined as the mean energy imparted by ionizing radiation to matter per unit mass. Formally expressed as

$$D = \frac{dE}{dm}, \quad (1.1)$$

where dE represents the energy deposited in a mass element dm . Absorbed dose quantifies the energy transfer that ultimately leads to biological response.

The International System of Units defines absorbed dose in gray (Gy), where 1 Gy equals 1 J/kg. This definition provides the basis for all subsequent dosimetric measurements and calculations. Clinical doses typically range from 1.8 to 2.0 Gy per fraction for conventional radiotherapy, with total treatment doses of 50–70 Gy delivered over 5–7 weeks.

1.3.1 Percent-depth-dose

Percent-depth-dose (PDD) curves represent one of the most fundamental characterizations in clinical electron beam dosimetry, describing how dose varies along the central axis of the beam as a function of depth in tissue. The PDD is defined as the ratio of dose at any depth to the dose at a reference depth, typically the depth of maximum dose, expressed as a percentage:

$$\text{PDD}(d) = \frac{D(d)}{D_{\max}} \times 100\%, \quad (1.2)$$

where $D(d)$ is the dose at depth d and $D_{\max}$ is the maximum dose in the depth-dose distribution.

1.3.2 Electron range

A charged particle carries its Coulomb field and consequently interacts electromagnetically with virtually every atom it encounters. Since each collision removes only a small fraction of the particle's kinetic energy, the energy loss process can be modeled using the continuous slowing-down approximation (CSDA), which treats energy loss as a continuous rather than discrete process. The CSDA range represents the mean path length that a particle with initial kinetic energy E_0 travels before coming to rest. This quantity can be expressed in terms of the total mass-stopping-power $S(E) = -dE/(\rho dx)$ (Section 1.4):

$$\mathcal{R}_{\text{CSDA}} = \int_0^{E_0} \left[-\frac{dE}{\rho dx} \right]^{-1} dE = \int_0^{E_0} S(E)^{-1} dE \quad [\text{g cm}^{-2}] \quad (1.3)$$

It is important to recognize that the CSDA range is purely a calculated quantity representing the mean path length along the electron's tortuous trajectory through the medium. This differs fundamentally from the depth of penetration in a defined direction, as electrons undergo multiple scattering events that cause their paths to deviate significantly from straight-line trajectories. The CSDA range therefore provides an

upper bound for the actual penetration depth in any given direction. Table 1.1 presents the CSDA range for electrons in water across various electron kinetic energies, demonstrating the energy dependence of this fundamental dosimetric parameter.

Table 1.1: CSDA ranges for electrons in water [1].

Electron energy (MeV)	$\mathcal{R}_{\text{CSDA}}$ (cm)
6	3.052
7	3.545
8	4.030
9	4.506
10	4.975
20	9.320
30	13.170

Beyond the CSDA range, clinical electron beam dosimetry employs several practical parameters that complement characterization of PDDs. Among these, the maximum range R_{max} is defined as the depth at which extrapolation of the tail of the curve meets the bremsstrahlung background, representing the largest penetration depth of electrons in the medium (Fig. 1.2). This parameter suffers from measurement imprecision due to statistical fluctuations in the tail region.

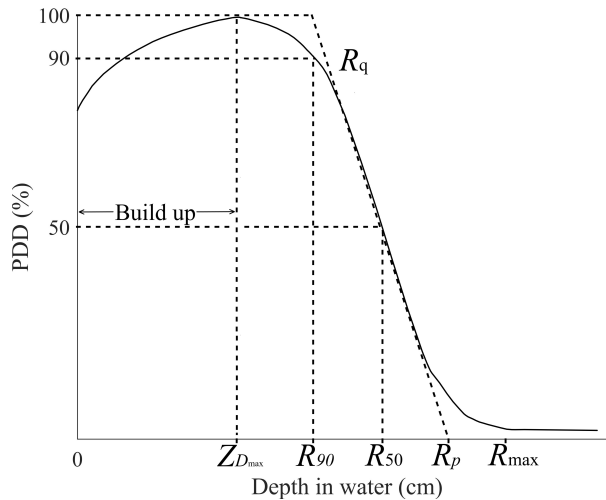


Figure 1.2: Typical electron beam PDD curve illustrating the definition of R_q , R_{90} , R_{50} , R_p and R_{max} [1].

Moreover, the practical range R_p represents the depth at which the beam's dose distribution falls off significantly after D_{max} , and is defined by the intersection between the tangent through the steepest portion of the depth-dose curve and the extrapolated bremsstrahlung background (Fig. 1.2). Additional parameters include R_{90} and R_{80} , corresponding to depths at 90% and 80% of maximum dose beyond $Z_{D_{\text{max}}}$, and the therapeutic range R_q which represents the depth where the tangent through the dose inflection point intersects the maximum dose level. The R_{50} parameter is especially significant, as it closely approximates the mean electron range, guides clinical energy selection based on target depth, and represents the beam quality index in electron beam dosimetry as defined by the International Atomic Energy Agency [2].

1.3.3 Electron beam energy specification

Due to the inherent complexity of the electron energy spectrum, an electron beam is typically described using the most probable energy at the phantom surface, E_0 , which is empirically correlated with the practical range R_p :

$$E_0 = 0.22 + 1.09R_p + 0.0025R_p^2 \quad (1.4)$$

Additionally, Table 1.2 presents the empirical correlations between E_0 and the characteristic ranges in water, and Table 1.3 provides representative electron depth-dose parameters as functions of nominal

beam energy. These parameters must be measured and verified for each electron beam before clinical implementation.

Table 1.2: Approximate ranges in water [3].

Parameter	Empirical relationship (cm)
R_{90}	$E_0/3.2$
R_{80}	$E_0/2.8$
R_{50}	$E_0/2.33$
R_p	$E_0/2$

Table 1.3: Typical depth dose parameters of electron beams in water [1].

Energy (MeV)	R_{90} (cm)	R_{80} (cm)	R_{50} (cm)	R_p (cm)	$\bar{E}_0$ (MeV)	Surface dose (%)
6	1.7	1.8	2.2	2.9	5.6	81
8	2.4	2.6	3.0	4.0	7.2	83
10	3.1	3.3	3.9	4.8	9.2	86
12	3.7	4.1	4.8	6.0	11.3	90
15	4.7	5.2	6.1	7.5	14.0	92
18	5.5	5.9	7.3	9.1	17.4	96

1.3.4 Build-up region

The region from the surface to $Z_{D_{\max}}$ is known as the build-up region, where secondary electrons are generated and increasingly contribute to the dose. The growing number and penetration depth of these secondary electrons causes the dose to climb until it peaks at $Z_{D_{\max}}$. For electron beams, this dose rise is much less pronounced than for megavoltage photon beams (Figs. 1.1 and 1.2), since electrons are both the primary and secondary dose carriers and begin depositing energy immediately upon entry.

An important feature of electron beams is their energy-dependent surface-dose behavior. Low-energy electrons scatter more readily and at wider angles, depositing most of their energy very near the surface. Consequently, the dose builds up rapidly, reaching its maximum at a shallow depth, resulting in a thin build-up region and a high surface-to- $D_{\max}$ ratio. Higher-energy electrons travel farther before significant scattering, so the dose increases more gradually over a deeper region. As a result, higher-energy beams exhibit a thicker build-up region and a lower surface-to- $D_{\max}$ ratio. Finally, $Z_{D_{\max}}$ generally increases with electron energy, as shown in Fig. 1.3.

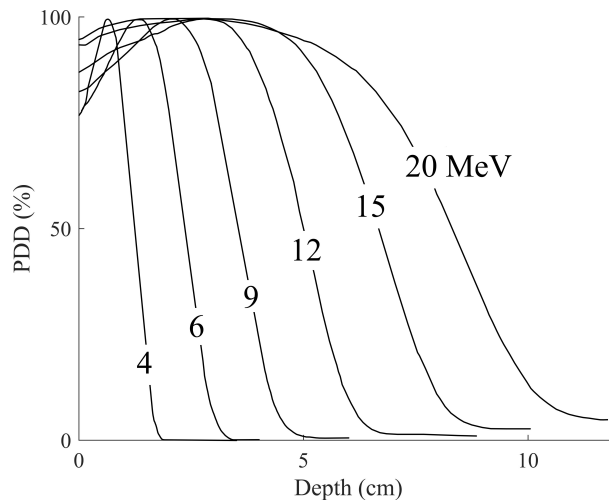


Figure 1.3: Central axis PDD curves for a family of electron beams from a high energy LINAC. All curves are normalized to 100% at $Z_{D_{\max}}$ [1].

1.3.5 Dose profiles

Lateral dose profiles, which plot normalized dose across the beam’s cross-section at a given depth (typically $Z_{D_{\max}}$), complement the PDD curves in fully characterizing an electron beam. They reveal four key parameters: field size, flatness, penumbra and symmetry. Field size refers to the beam’s geometric dimensions at the phantom surface, usually defined by the 50% isodose line. Flatness is the maximum dose variation across the central 80% of the field, while symmetry measures dose differences at equal lateral distances from the central axis, both remaining within a few percent. Penumbra denotes the lateral distance over which dose falls from 80% to 20% of the central-axis value, typically 1–2 cm at therapeutic depths (Fig. 1.4).

Together, PDD and lateral-profile data provide a comprehensive map of depth and transverse dose gradients essential for precise electron beam therapy.

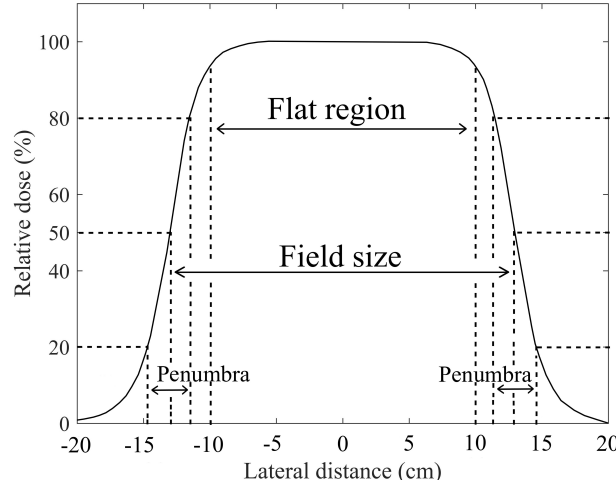


Figure 1.4: Profile curve illustrating the definition of field size, penumbra and flatness.

1.4 Electron interactions

Charged-particle interactions with matter via the Coulomb force encompass several processes that together produce the complex energy-deposition patterns observed in clinical dosimetry. These processes can be classified as elastic scattering—where electrons change direction with negligible energy loss—and inelastic collisions—where significant energy is transferred to the absorbing medium. The character of any given interaction may be further understood through the ratio of the classical impact parameter b to the atomic radius a , as illustrated in Fig. 1.5.

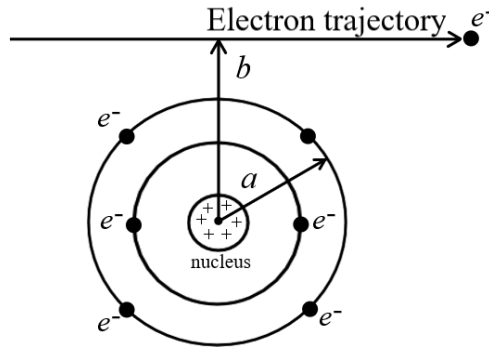


Figure 1.5: Key parameters in charged-particle collisions with atoms: a is the classical atomic radius and b is the impact parameter [4].

1.4.1 Elastic scattering

When $b \gg a$, electrons undergo “soft” collisions with the atom, producing only slight deflections and negligible energy loss. At therapeutic energies, the cumulative effect of these soft events gives rise to *multiple Coulomb scattering*, which gradually broadens the beam and alters its angular distribution.

The probability of scattering at an angle θ is given by the following term in the spinless Rutherford model differential cross section:

$$\frac{d\sigma}{d\Omega} \propto \frac{1}{\sin^4(\theta/2)}, \quad (1.5)$$

where $d\sigma/d\Omega$ is the effective area for scattering into the solid angle $d\Omega$, and σ has units of area, representing the “target size” of the atom.

If $\theta \approx 0$, then $d\sigma/d\Omega \approx 16/\theta^4$ and $d\sigma/d\Omega \rightarrow \infty$, meaning that nearly every electron interacting with the Coulomb field is deflected, even if only by a very small angle. On the other hand, if $\theta = \pi$, then $d\sigma/d\Omega \rightarrow 1$, meaning that the probability of an electron being backscattered is minimal.

1.4.2 Ionization and excitation

When $b \approx a$, the incoming electron penetrates close enough to an individual orbital electron to transfer sufficient energy for ionization, producing δ rays—high-energy secondary electrons that travel away from the primary track. Slightly larger impact parameters still produce excitations without full ionization. Both processes contribute to the *collision stopping power*, which quantifies the average energy loss per unit path length and is given by the Bethe–Bloch formula:

$$-\frac{dE}{dx}\Big|_{\text{col}} = \frac{2\pi e^4 N Z z^2}{m_e v^2} \left[\ln\left(\frac{2m_e c^2 \beta^2}{(1 - \beta^2)I}\right) - \beta^2 \right]. \quad (1.6)$$

where, $-dE/dx|_{\text{col}}$ is the mean energy loss by unit of length by collisions with atomic electrons; N is the number density of atoms in the medium, v is the electron’s speed and $\beta = v/c$. The mean excitation energy I represents the average of all excitation and ionization energies of the medium.

1.4.3 Radiative energy loss

When $b \ll a$, the electron passes very close to the nucleus, emitting bremsstrahlung photons with energies up to the original electron energy. The probability of this radiative process scales roughly as $Z^2 E$. The corresponding radiative stopping power is

$$-\frac{dE}{dx}\Big|_{\text{rad}} = \frac{E}{X_0}, \quad (1.7)$$

where X_0 is the material’s radiation length. In tissue-equivalent media, these losses are minor below 10 MeV but can contribute 10–15 % of the total at 20 MeV.

The total stopping power is then the sum of collision and radiative components:

$$-\frac{dE}{dx} = -\frac{dE}{dx}\Big|_{\text{col}} - \frac{dE}{dx}\Big|_{\text{rad}}. \quad (1.8)$$

1.5 Photon interactions

Although this work focuses primarily on electron beams, bremsstrahlung and the resulting mixed fields in clinical practice demand a parallel treatment of photon interactions. At megavoltage energies, three processes dominate: photoelectric absorption, Compton scattering, and pair production (Fig 1.6).

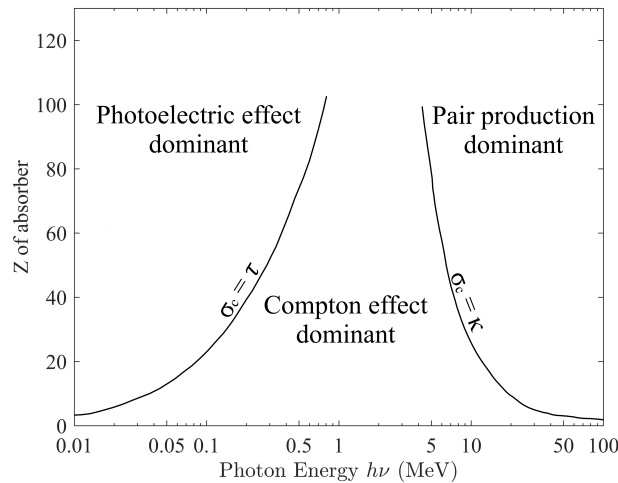


Figure 1.6: Regions of dominance for photoelectric absorption (τ), Compton scattering (σ_c), and pair production (κ) in the Z – E_γ plane [4].

At low photon energies (or high Z), the photoelectric effect prevails until the curve $\sigma_c = \tau$ is crossed, beyond which Compton effect dominates. As energy increases further, the pair-production threshold is reached and, above the curve $\sigma_c = \kappa$, pair production becomes the primary attenuation mechanism. The megavoltage range and soft-tissue Z values used in clinical radiotherapy fall squarely in the Compton-dominant region, whereas photoelectric absorption remains significant in bone or contrast-enhanced tissues at a few hundred keV, and pair production governs interactions in high- Z beam-forming components at multi-MeV energies.

1.5.1 Photoelectric effect

When an incident photon transfers its entire energy to a bounded electron, the ejected photoelectron departs with kinetic energy:

$$T = E_\gamma - E_b, \quad (1.9)$$

where E_γ is the photon energy and E_b the electron's binding energy (Fig. 1.7). Following ejection, the atom relaxes via characteristic X-ray emission, which further contribute to local dose deposition.

The probability of a photoelectric event is described by the cross section τ , which scales approximately as

$$\tau \propto \frac{Z^4}{E_\gamma^3}, \quad (1.10)$$

so that absorption is strongest at low E_γ and in high- Z media (Fig. 1.6).

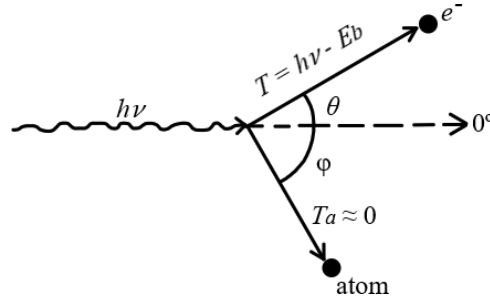


Figure 1.7: Kinematics of the photoelectric effect. A photon of energy $E_\gamma = h\nu$ is absorbed by an inner-shell electron (binding energy E_b), which is ejected with kinetic energy T . The residual atom recoils (negligible kinetic energy) to conserve momentum [4].

1.5.2 Compton scattering

When photons interact predominantly with “free” electrons, they suffer inelastic scattering, transferring part of their energy to recoil electrons and changing direction (Fig. 1.8). As the photon loses energy in the collision, its wavelength increases according to $\lambda = hc/E$, so lower-energy scattered photons emerge with longer wavelengths.

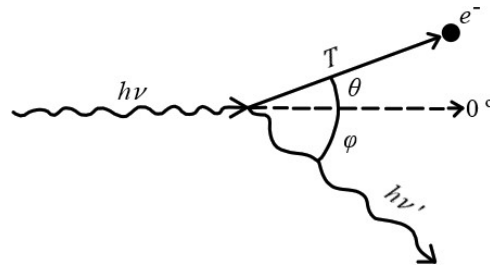


Figure 1.8: Energy in the Compton effect. A photon of quantum energy $h\nu$ incident from the left strikes an unbound stationary electron, scattering it at angle θ relative to the incident opposite side of the original direction, in the same scattering plane. Energy and momentum are each conserved [4].

The energy E' of the scattered photon at angle θ is then given by:

$$E' = \frac{h\nu}{1 + \frac{h\nu}{m_e c^2} (1 - \cos \theta)}; \quad (1.11)$$

and the differential cross section follows the Klein–Nishina formula:

$$\frac{d\sigma_c}{d\Omega} = r_e^2 \left(\frac{E'}{E_\gamma} \right)^2 \left(\frac{E_\gamma}{E'} + \frac{E'}{E_\gamma} - \sin^2 \theta \right), \quad (1.12)$$

where r_e is the classical electron radius. The recoil electron leaves with kinetic energy

$$T_e = E_\gamma - E', \quad (1.13)$$

and deposits dose locally along its short track. Because the integrated Compton cross section σ_c scales roughly with electron density (i.e. nearly independent of Z in soft tissue), Compton scattering governs attenuation and scatter in megavoltage beams, shaping both depth-dose curves and lateral dose profiles.

1.5.3 Pair production

Pair production is an absorption process in which a photon disappears and gives rise to an electron and a positron. It can only occur in a Coulomb force field, usually that near an atomic nucleus. Thus two electrons and a positron are ejected from the site of the interaction (Fig. 1.9).

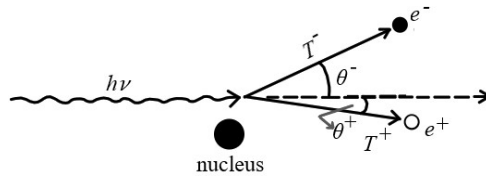


Figure 1.9: Pair production in the Coloumb force field of an atomic nucleus. An incident photon of quantum energy $h\nu$ vanishes, giving rise to a positron-electron pair. The atom participates in conservation of momentum, but receives negligible kinetic energy [4].

A minimum photon energy of $2m_e c^2 = 1.022$ MeV is required for conversion into an electron-positron. Above this threshold, the pair-production cross section κ increases approximately as:

$$\kappa \propto Z^2 \ln\left(\frac{2E_\gamma}{m_e c^2}\right), \quad (1.14)$$

making it more probable in high- Z materials (e.g. tungsten or lead). The resulting electron and positron share the excess photon energy and emerge with a pronounced forward bias. The positron eventually annihilates with an ambient electron, emitting two 0.511 MeV photons in opposite directions; these travel centimeters in tissue, creating a low-gradient dose “tail”. Accurate Monte Carlo transport therefore requires tracking both the initial pair kinematics and the subsequent annihilation photons.

1.6 Why use Monte Carlo?

The complexity of radiation transport in heterogeneous media, combined with the stochastic nature of individual particle interactions, makes analytical solutions intractable for most clinically relevant geometries. Traditional dose calculation algorithms employ various approximations—such as assuming uniform density, neglecting scatter from outside the primary beam, or using simplified interaction cross-sections—that compromise accuracy in complex situations. MC methods address these limitations by simulating the fundamental physics of particle transport without the approximations inherent in analytical approaches.

MC techniques model radiation transport by following individual particles through stochastic sampling of interaction probabilities, energy loss mechanisms, and angular distributions (Fig. 1.10). Each particle history represents one possible realization of the transport process, with the simulation’s accuracy improving as the number of histories increases according to $1/\sqrt{N}$ statistical convergence. This approach naturally accounts for all relevant physical processes—multiple scattering, energy straggling, secondary particle production, and complex geometries—providing highly accurate dose predictions when sufficient statistics are accumulated.

The fundamental advantage of MC methods lies in their ability to model the complete transport physics without geometric or material approximations. Unlike analytical algorithms that must assume homogeneous media or make simplifying assumptions about scatter contributions, MC simulations track particles through arbitrarily complex three-dimensional geometries with heterogeneous material distributions. This capability makes MC methods particularly valuable for treatment planning in anatomically realistic patient models where tissue heterogeneities significantly influence dose distribution.

For electron beam dosimetry specifically, MC methods excel at modeling the complex multiple scattering processes that dominate electron transport. Analytical algorithms typically employ continuous slowing-down approximations (Section 1.3.2) or multiple-scattering theories that, while computationally efficient, cannot fully capture the detailed physics of electron track structure. MC simulations explicitly sample from angular and energy distributions, accurately reproducing the lateral dose spreading, range straggling, and backscatter effects observed experimentally.

The stochastic nature of MC simulations requires careful attention to statistical uncertainties and convergence criteria. Each simulation provides an estimate of the true dose value with associated statistical uncertainty that decreases as more particle histories are accumulated. This statistical uncertainty provides quantitative confidence intervals for simulation results. For clinical applications, uncertainties below 2-3% are generally required for adequate precision.

Modern MC codes like *Geometry and Tracking 4* (GEANT4) or *Electron Gamma Shower* (EGS) incorporate decades of cross-section measurements and transport algorithm development, providing validated physics models for therapeutic energy ranges.

Computational requirements represent the primary limitation of MC methods, with simulation times often orders of magnitude longer than analytical calculations. However, advances in computer hardware, parallel processing, and variance reduction techniques have dramatically reduced these barriers. Modern workstations can perform detailed MC calculations that required supercomputers just decades ago, making these methods accessible for routine research and increasingly for clinical applications (Section 2.2).

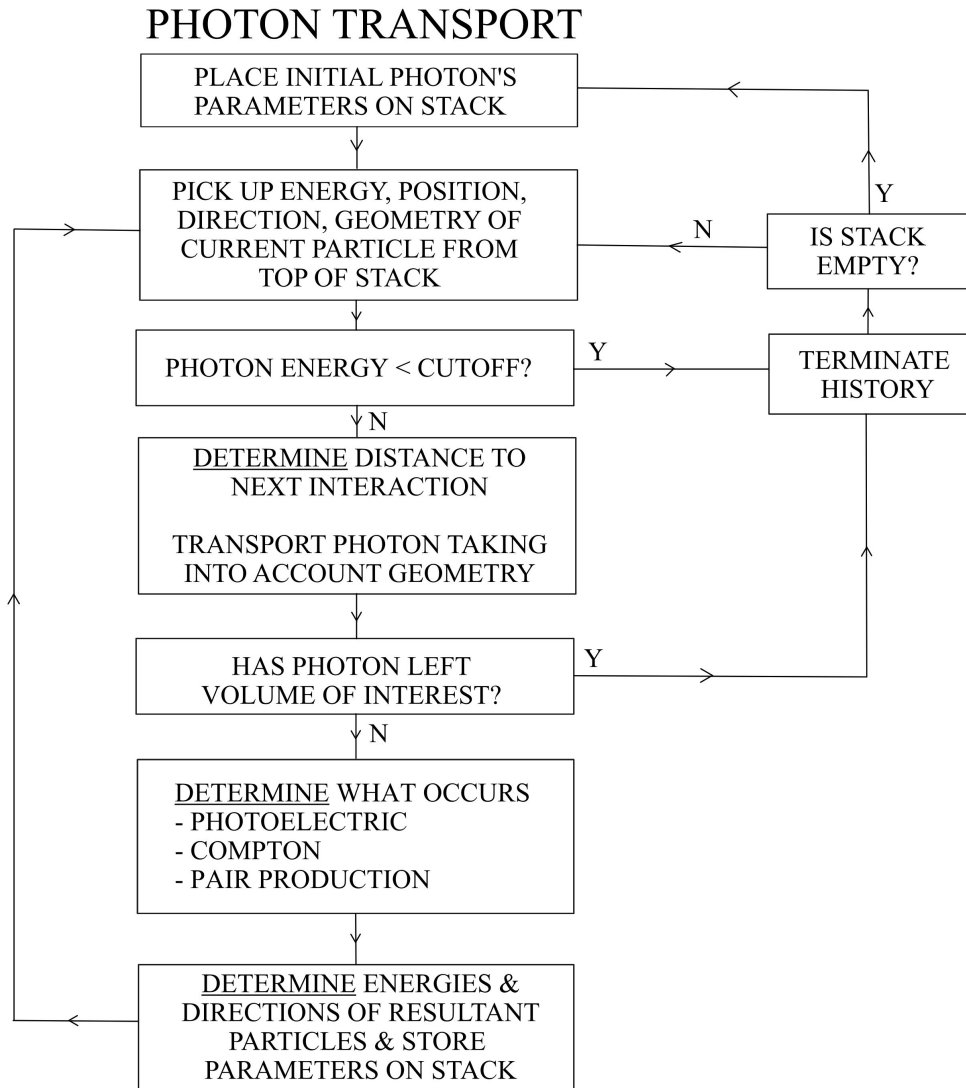


Figure 1.10: Logic flow of a Monte Carlo simulation of photon transport [5].

II Materials and methods

This chapter provides a description of the computational tools, physical models, and simulation setup used in this work. The software frameworks and hardware environment are introduced, followed by a concise presentation of the chosen *physics list*. The phantom geometry, source configuration, material definitions, and resolution are detailed. The approximation method is outlined, and the formulas used for dose calculation and uncertainty estimation are presented.

2.1 GEANT4 & GATE

The GEANT4 program provides a versatile, object-oriented C++ framework for modeling particle transport and interactions in matter. Its modular architecture enables precise definition of complex geometries, selection of tailored physics processes, detailed particle tracking, and customizable result recording. This flexibility underpins GEANT4’s widespread adoption, but it also requires users to master its class hierarchy, configure appropriate physics lists, and navigate the build system.

Originally developed by the *European Organization for Nuclear Research* (CERN) for high-energy physics, GEANT4’s architecture quickly garnered interest from medical physicists aiming to simulate radiation transport in imaging and therapy. However, implementing custom C++ classes to define detectors or scoring routines can pose a steep learning curve. To mitigate this challenge, several domain-specific frameworks have been built atop GEANT4, providing higher-level interfaces, ready-to-use physics lists, and modular components tailored to clinical and preclinical applications.

One prominent example is the GATE toolkit, which was employed in this study. Rather than writing C++ code, users define scanner geometry, source characteristics, acquisition protocols, and output formats through scriptable macros. GATE offers preconfigured digitizer chains, dynamic motion handling, and tools for importing patient or phantom datasets, enabling medical researchers to concentrate on experimental design rather than low-level implementation details (Fig. 2.1). Nonetheless, effective use of GATE still requires a firm grasp of radiation physics: choosing suitable physics lists, benchmarking simulations against experimental data, and managing statistical uncertainties and computational performance. Developed by the OpenGATE collaboration since the early 2000s, GATE was first released publicly by Jan et al. in 2004 [6].

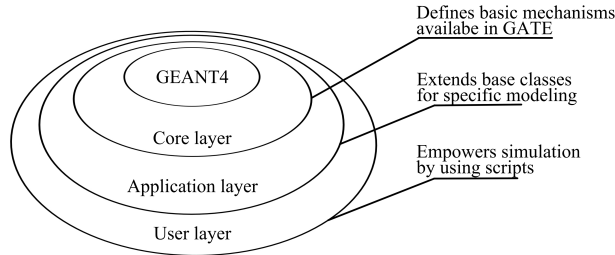


Figure 2.1: Illustration of the GATE layered structure [6].

2.2 Software and hardware specifications

All simulations presented in this work were executed on a single HP Laptop 15s-fq4xxx with the following specifications:

- **CPU:** 11th Gen Intel® Core™ i5-1155G7 (4 physical cores, 8 threads), base clock 2.50 GHz (turbo up to 4.50 GHz).
- **RAM:** 32.0 GB DDR4.
- **Storage:** Samsung Portable SSD T7 1 TB.
- **GPU:** Intel® Xe Graphics (integrated).
- **Operating System:** Ubuntu 22.04.5 LTS, 64-bit Linux kernel 5.15.
- **GEANT4:** version 11.2.1, built from source.
- **GATE:** version 9.4.1, configured against GEANT4 11.2.1.

Although GATE offers multi-threading capabilities, both GEANT4 and GATE were compiled in single-threaded mode to avoid the additional build complexity and time. While parallel event processing can markedly reduce CPU runtimes by leveraging multiple cores, all simulations presented here therefore executed on a single thread. Likewise, despite the availability of GPU offloading—particularly beneficial on discrete graphics hardware—no GPU acceleration was employed on this integrated Intel® Xe system.

All post-processing tasks—including data parsing, statistical analysis, and figure generation—were performed in GNU Octave 6.4.0. Three-dimensional inspection of detector geometries and particle tracks utilized GEANT4’s OGLIQt interface, which opens an interactive OpenGL-based Qt window for model visualization. Final illustrations and annotated schematics were composed in MS Paint 3D and Krita.

2.3 Physics list

The selection of a *physics list* in both GATE and its GEANT4 core dictates which fundamental processes—electromagnetic, hadronic, decay, optical, and so on—are included and how they are treated. Its configuration is pivotal for striking a balance between physical accuracy and computational cost: a “rich” list that incorporates detailed low-energy models and tight step restrictions enhances the fidelity of dose and track-structure predictions but incurs longer runtimes and higher memory demands.

Several preconfigured electromagnetic physics lists are widely adopted in medical-physics simulations:

- **emstandard**: the default intermediate-energy package (keV–GeV) available in three variants (**opt1**, **opt2**, **opt3**).
- **emlivermore**: based on Livermore libraries, modeling detailed atomic-shell processes and fluorescence down to a few hundred eV.
- **empenelope**: implements PENELOPE Monte Carlo models for photons and electrons, improving accuracy from sub-keV to tens of MeV.

In this work, **emstandard_opt3** was selected for its refined electron-transport algorithm, which balances fidelity and computational speed at therapy energies (6–21 MeV). It encompasses all standard electromagnetic processes.

2.4 Simulation setup

It’s crucial to recognize that a trustworthy simulation rests on two pillars: a clear definition of the experiment’s geometry and a precise description of its source. The geometry outlines the shapes, sizes, positions, and materials of all components, carving out the exact physical environment where particles will travel and interact. At the same time, the source settings—particle type, energy spectrum, beam profile, and angular spread—establish the starting conditions for each simulated event.

2.4.1 World

In GEANT4’s simulation “philosophy”, a *world*—the outermost volume that defines the material and spatial boundaries within which all geometry and particle interactions occur—must be defined. Its dimensions and material selection require careful consideration: enlarging the world enhances the capture of wide-angle scatter and backscattered particles but incurs greater CPU and memory demands, whereas downsizing risks prematurely truncating tracks and skewing results. Therefore, the world should be sufficiently large to encompass all relevant physics while remaining compact enough to optimize performance.

In this work, the world is implemented as a $1 \times 1 \times 3 \text{ m}^3$ air-filled box. All other volumes—such as the water-box phantom and any auxiliary scoring or shielding components—are placed hierarchically as “daughter” volumes. This organization simplifies overlap checks and ensures efficient geometry construction.

2.4.2 Phantom

In radiotherapy, simulations and measurements frequently utilize what is known as a “phantom”—a physical or virtual representation of human tissue that replicates its interaction with ionizing radiation. A phantom can be conceptualized as an object composed of tissue-equivalent materials, designed with human-like dimensions, and employed to characterize dose distributions under controlled and reproducible conditions.

Given that the adult human body is composed of approximately 60–70% water, a homogeneous water volume serves as a reliable surrogate for dosimetric purposes, avoiding the confounding effects introduced by tissue heterogeneities. Water’s mass density, electron density, and effective atomic number are closely matched to those of soft tissue, resulting in photon and electron attenuation and scattering behaviors that effectively mimic those observed in human tissue (Fig. 2.2).

In the present work, the phantom is a $45 \times 45 \times 40 \text{ cm}^3$ block of pure water, centered at $(0, 0, 20 \text{ cm})$. Its front surface therefore lies at $Z = 0$ and its geometric center at $Z = 20 \text{ cm}$. An electron beam travels along the $+Z$ axis from a point source located at $Z = -100 \text{ cm}$, giving a source-to-surface distance (SSD) of 100 cm and a source-to-phantom-center distance of 120 cm . This arrangement, commonly used in clinical electron therapy (Figs. 2.2b and 2.3), streamlines geometry definition while conforming to standard depth-dose measurement protocols.

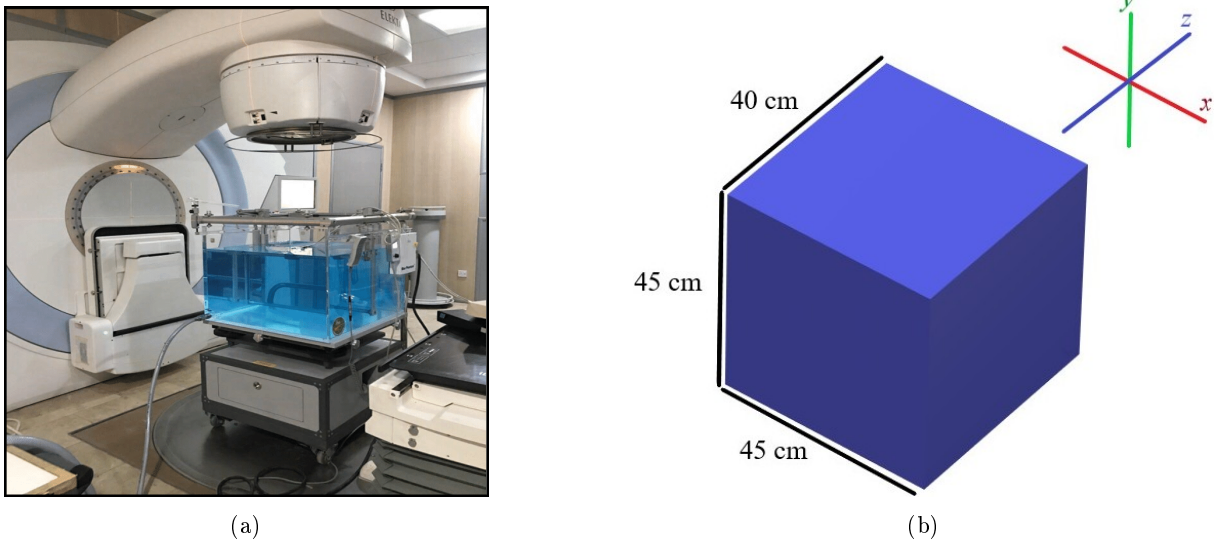


Figure 2.2: (a) Example of a water-tank phantom used for depth-dose measurements in electron beam dosimetry, under the gantry of a LINAC [8]; (b) Schematic representation of the homogeneous water-box phantom, showing its dimensions of $45 \times 45 \times 40 \text{ cm}^3$.

2.4.3 Production cuts

Production cuts define the minimum range below which secondary electrons, positrons, and photons are not explicitly tracked but instead deposit their energy locally. Here, a 10 mm cut was applied in the air-filled world and a 0.1 mm cut within the water phantom. The coarse air cut omits low-range electrons with negligible impact on phantom dose, reducing computational load, while the fine water cut preserves accurate modeling of low-energy secondaries in the build-up and distal fall-off regions. Thoughtful selection of these thresholds thus maintains dosimetric fidelity without unduly extending CPU time.

2.4.4 Scoring

Scoring in GATE is implemented through modular entities known as “actors.” Each actor attaches to a designated volume or surface within the geometry and autonomously tallies chosen physical observables as particles interact in that region. Actor types are specialized for distinct tasks—some extract electromagnetic parameters, others compute regional dose distributions, and still others generate differential spectra of deposited energy—allowing flexible customization of simulation outputs.

In this study, two main actors were employed: the **DoseActor** to record depth-dose curves, lateral dose profiles, and their uncertainties, and the **EnergySpectrumActor** to produce energy deposition spectra. The **DoseActor**, as the core of this work, records energy deposition within a user-defined volume of the phantom. For depth-dose scoring, that volume is a $0.5 \times 0.5 \times 40 \text{ cm}^3$ prism centered on the beam axis and subdivided into 200 slices along z . Each slice acts as a virtual detector of volume $0.5 \times 0.5 \times 0.2 \text{ cm}^3$, accumulating energy deposits to produce a dose-depth curve sampled at 200 points (0.2 cm resolution) (Fig. 2.3a). For clarity, each of these slices will hereafter be called a *dosel*—a slice of the **DoseActor**’s scoring prism that registers the local dose value.

For the lateral x profile, the **DoseActor** is applied to a $45 \times 1 \times 1 \text{ cm}$ prism positioned at $Z_{D_{\max}}$, the depth of maximum dose for each energy. Positioning the scoring plane at $Z_{D_{\max}}$ avoids variations in the build-up region and aligns with clinical beam-flatness measurement protocols. The 45 cm span is segmented into 64 dosels of 0.7 cm each. Due to the beam’s cylindrical symmetry, the lateral y profile mirrors that of x and was thus omitted.

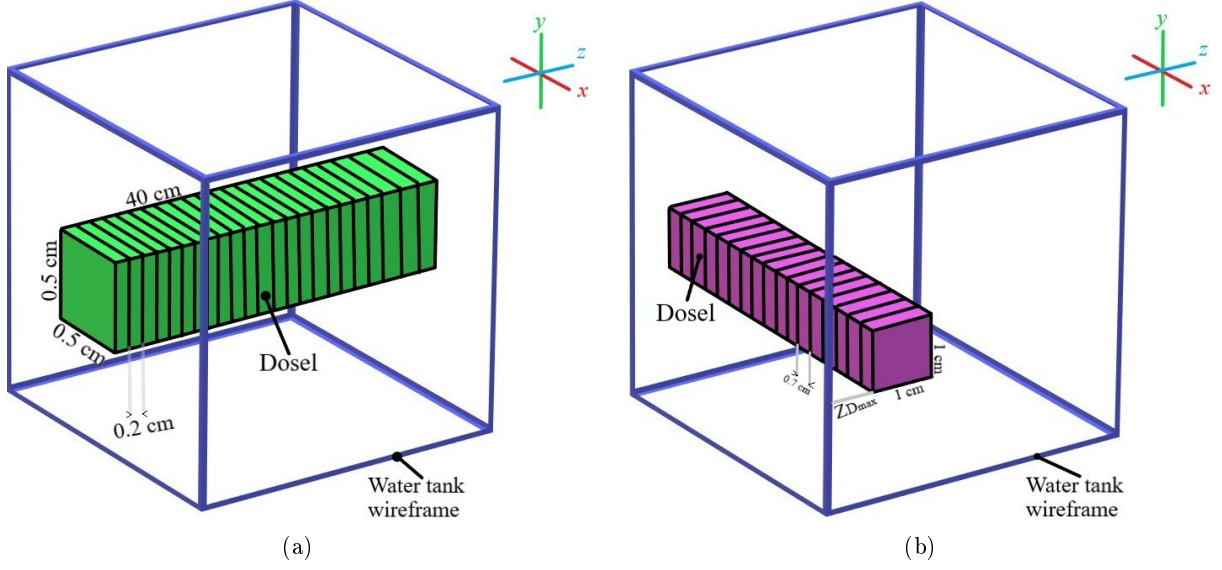


Figure 2.3: Dose scoring volumes within the phantom: (a) depth-dose detectors of $0.5 \times 0.5 \times 0.2 \text{ cm}^3$ arrayed along the central axis; (b) lateral dose detectors of $0.7 \times 1 \times 1 \text{ cm}^3$ arranged at $Z_{D_{\max}}$.

2.4.5 Source

Lastly, the primary electron source is defined using GATE’s General Particle Source (GPS) as a point emitter located at $(0, 0, -100 \text{ cm})$, establishing a SSD of 100 cm. The beam is confined to a 2.86° half-angle cone, mimicking clinical collimation and delivering uniform irradiation across the phantom face. Four monoenergetic beams at 6, 10, 18 and 21 MeV were simulated to span the clinical therapeutic range. Each source emits particles uniformly over the full $0\text{--}360^\circ$ azimuth to preserve cylindrical symmetry, and 10^8 primaries per energy were generated to achieve better than 2% statistical uncertainty within acceptable runtimes.

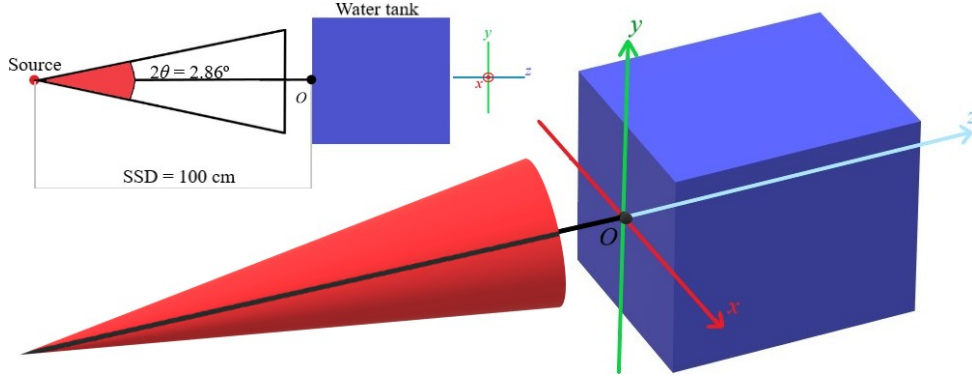


Figure 2.4: 2D inset and 3D rendering of the beam–phantom setup: a point-source electron beam at $(0, 0, -100 \text{ cm})$ forms a 2.86° cone toward a water phantom centered at $(0, 0, 20 \text{ cm})$ (SSD = 100 cm). The conical depiction is purely schematic, omitting actual beam divergence and electron scattering in air.

2.5 Dose scoring and uncertainty quantification

Accurate Monte Carlo dose scoring requires converting discrete energy deposits into both a dose estimate and its associated statistical uncertainty. In GATE, this is handled by the `DoseActor`, which records the necessary quantities and computes the uncertainty.

2.5.1 Dose scoring

Imagine an electron e_i^- traversing dosel j . As it interacts with the medium, it deposits energy $e_{i,j}$ within that dosel. Each dosel is a rectangular prism of volume V_j and mass $m_j = \rho V_j$, where ρ is the phantom density. The dose deposited by a single event is then:

$$d_{i,j} = \frac{e_{i,j}}{m_j}. \quad (2.1)$$

Therefore, summing over all N particles yields the total dose in dosel j :

$$D_j = \frac{1}{m_j} \sum_{i=1}^N e_{i,j} = \sum_{i=1}^N d_{i,j} \quad (2.2)$$

2.5.2 Dose uncertainty

By the König–Huygens theorem, the variance of a random variable X can be written as

$$\text{Var}[X] = \overline{X^2} - (\overline{X})^2,$$

where $\overline{X} = \frac{1}{N} \sum_{i=1}^N X_i$ is the first moment (arithmetic mean) and $\overline{X^2} = \frac{1}{N} \sum_{i=1}^N X_i^2$ is the second moment. The standard deviation $\sigma = \sqrt{\text{Var}[X]}$ quantifies the data spread, and the standard error of the mean $\sigma_{\overline{X}} = \sigma/\sqrt{N}$ indicates how the uncertainty in the sample mean decreases with increasing N .

With these definitions in hand, one can express the first and second moments of the total dose in the dosel. Since D_j is the sum of all energy deposits, (Eq. 2.2), the mean dose becomes:

$$\overline{D_j} = \frac{1}{N} \frac{1}{m_j} \sum_{i=1}^N e_{i,j} = \frac{1}{N} \sum_{i=1}^N d_{i,j}; \quad (2.3)$$

and the second moment:

$$\overline{D_j^2} = \frac{1}{N} \frac{1}{m_j^2} \sum_{i=1}^N e_{i,j}^2 = \frac{1}{N} \sum_{i=1}^N d_{i,j}^2. \quad (2.4)$$

In statistics, the term $\sum_{i=1}^N d_{i,j}$ and $\sum_{i=1}^N d_{i,j}^2$ are usually called the raw first and second moment. For simplification, let's call this sums S_1 and S_2 , respectively. Henceforth, the equations (2.3) and (2.4) can be rewritten as:

$$\overline{D_j} = \frac{S_1}{N}, \quad (2.5)$$

$$\overline{D_j^2} = \frac{S_2}{N}, \quad (2.6)$$

and, finally, the standard error of the mean—or absolute dose uncertainty—is given by:

$$\sigma_{\overline{D_j}} = \frac{\sqrt{S_2/N - (S_1/N)^2}}{\sqrt{N}} \text{ [Gy]}. \quad (2.7)$$

However, since the goal is to quantify dose precision rather than absolute value, relative uncertainty in dosel j offers a more direct comparison across depth or beam setting.

Dividing the standard error from Equation (2.7) by the mean dose in Equation (2.5) yields a dimensionless metric, defined as U_j :

$$U_j = \frac{\sigma_{\overline{D_j}}}{\overline{D_j}} = \frac{\sqrt{S_2/N - (S_1/N)^2}}{(S_1/N) \sqrt{N}} = \sqrt{\frac{S_2}{S_1^2} - \frac{1}{N}}, \quad (2.8)$$

and because this work focuses on percent-depth-dose, one must do the following error propagation:

$$\Delta\text{PDD}_j(\%) = \text{PDD}_j \sqrt{U_j^2 + U_{\max}^2} \times 100\% \quad (2.9)$$

where U_j is the relative uncertainty in the j th dosel and $U_{\max}$ is the largest single-dosel uncertainty.

Multiplying the root-sum-square of these components by the local PDD yields a rigorous estimate of the percentage uncertainty at each depth. This relative standard error is especially important in GATE simulations using the **DoseActor**, as it indicates whether enough particle histories have been simulated to yield statistically reliable dose values.

2.6 Problem statement

In this work, the objective is to determine central-axis depth-dose curves, profiles and their relative statistical uncertainties for clinical electron beams simulated with GATE's **DoseActor**. Under the conditions defined in this chapter, 10^8 electrons are emitted from each of four monoenergetic point sources (6, 10, 18 and 21 MeV) located 100 cm upstream along the Z -axis. Particles traverse the air column, enter a $45 \times 45 \times 40 \text{ cm}^3$ water phantom centered at $Z = 20 \text{ cm}$, and continue until exiting or depositing all their energy. Central-axis depth-dose values are scored in 200 dosels of $0.5 \times 0.5 \times 0.2 \text{ cm}^3$, while lateral profiles are scored in 64 dosels of $0.7 \times 1 \times 1 \text{ cm}^3$. Dose and relative uncertainty are then computed via Equations (2.2) and (2.9), providing the basis for the results presented in Chapter III.

III Results and discussion

All results in this chapter were generated using the configuration and scoring parameters of Section 2.6. To enable precise control over individual dosimetric endpoints, PDD curves and dose profiles were generated in separate simulations rather than via a single multi-task run; CPU time is accordingly reported alongside each result. A detailed interpretation of these findings is presented in Section 3.4.

3.1 Percent-depth-dose curves

6 MeV

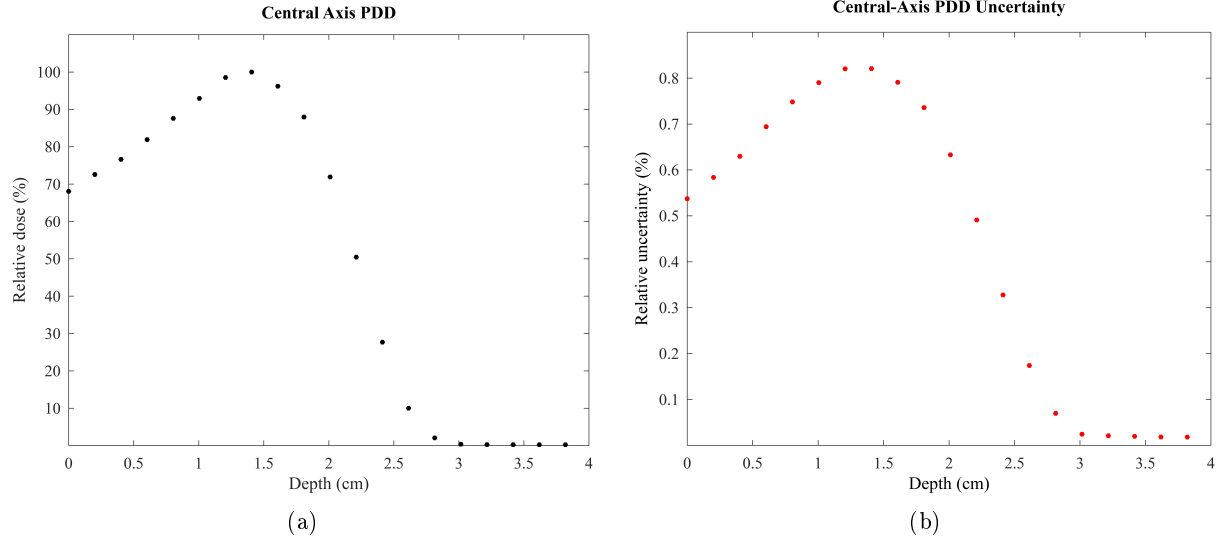


Figure 3.1: (a) Central-axis PDD for a water phantom irradiated with a 6 MeV electron beam, scored with 200 dosels at 0.2 cm resolution. Dosimetric parameters are $Z_{D\max} = 1.4$ cm, $R_{90} = 1.8$ cm, $R_{80} = 1.9$ cm, $R_{50} = 2.2$ cm and $R_p = 2.7$ cm. (b) Relative uncertainty in the PDD, remaining below 0.8% across the entire curve. Note the slight inflection around 1 cm—reflecting the initial build-up dynamics unique to low-energy electrons. Total CPU time: 2h 33min 20s.

10 MeV

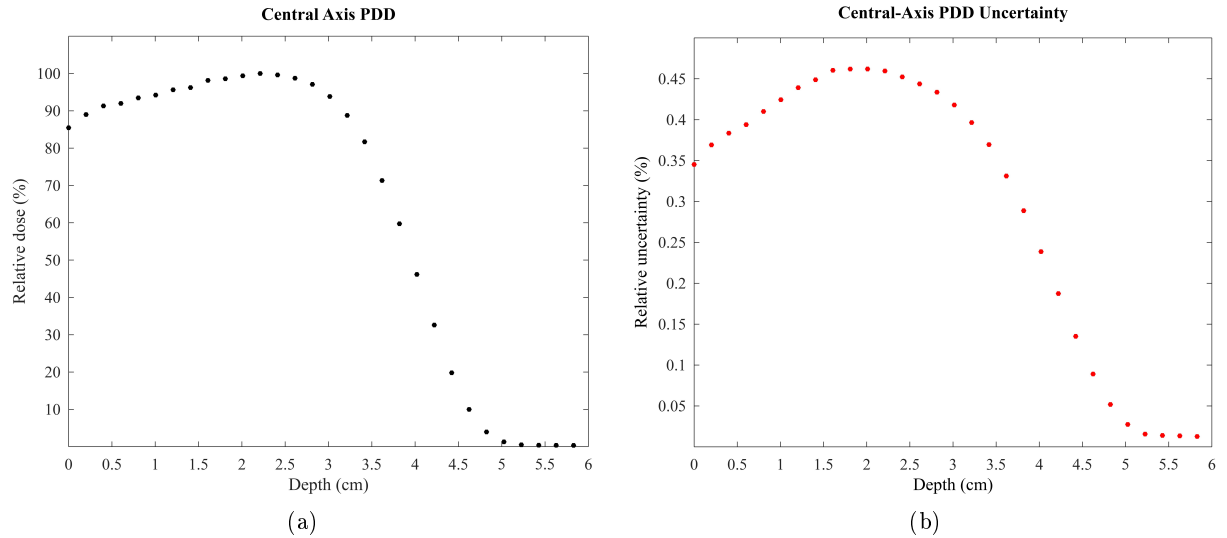


Figure 3.2: (a) Central-axis PDD for a water phantom irradiated with a 10 MeV electron beam, scored with 200 dosels at 0.2 cm resolution. Dosimetric parameters are $Z_{D\max} = 2.2$ cm, $R_{90} = 3.2$ cm, $R_{80} = 3.4$ cm, $R_{50} = 4.0$ cm and $R_p = 4.7$ cm. (b) Relative uncertainty in the PDD, remaining below 0.5% across the entire curve. Total CPU time: 4h 42min 25s.

18 MeV

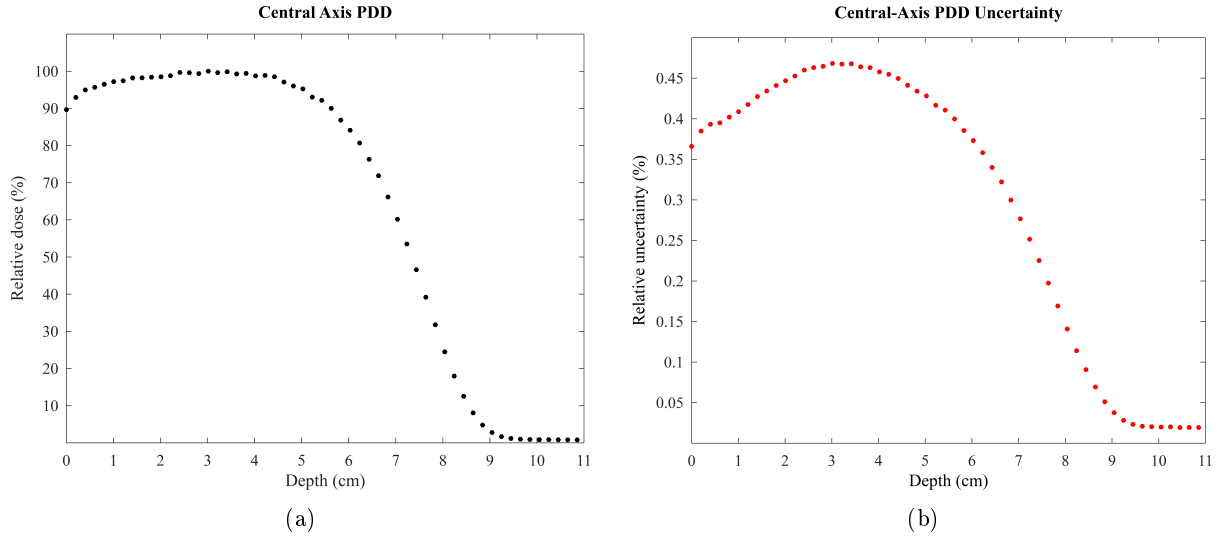


Figure 3.3: (a) Central-axis PDD for a water phantom irradiated with an 18 MeV electron beam, scored with 200 dosels at 0.2 cm resolution. Dosimetric parameters are $Z_{D\max} = 3.0$ cm, $R_{90} = 5.6$ cm, $R_{80} = 6.3$ cm, $R_{50} = 7.3$ cm and $R_p = 9.1$ cm. (b) Relative uncertainty in the PDD, held below 0.5% across the curve. The extended plateau slope is notably flatter, illustrating the beam's quasi-uniform energy deposition before dose fall-off. Total CPU time: 5h 8min 48s.

21 MeV

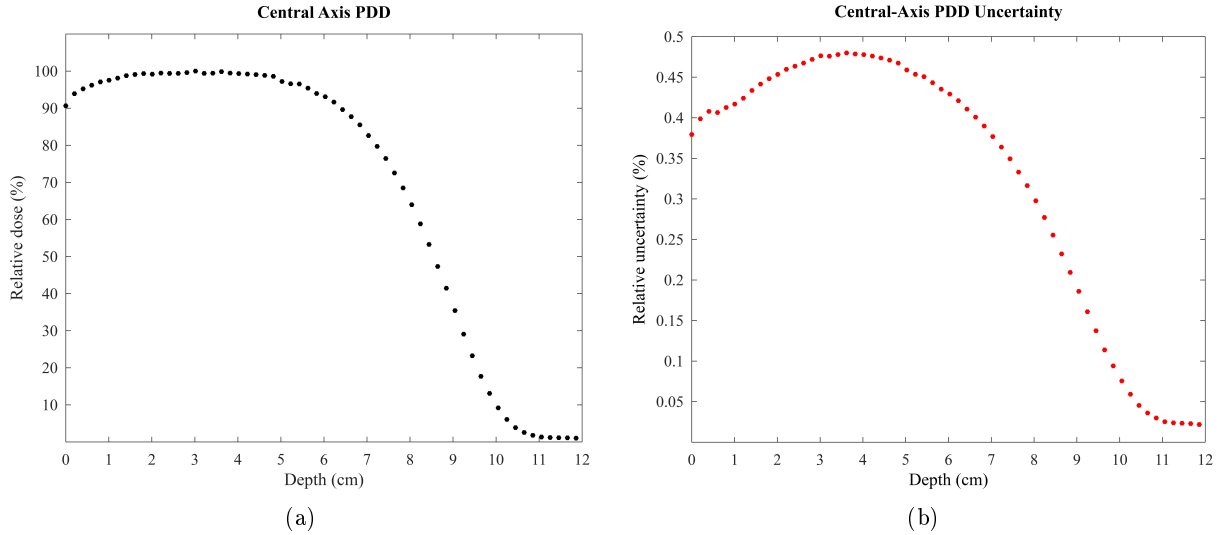


Figure 3.4: (a) Central-axis PDD for a water phantom irradiated with a 21 MeV electron beam, scored with 200 dosels at 0.2 cm resolution. Dosimetric parameters are $Z_{D\max} = 3.02$ cm, $R_{90} = 6.4$ cm, $R_{80} = 7.2$ cm, $R_{50} = 8.6$ cm and $R_p = 10.1$ cm. (b) Relative uncertainty in the PDD, below 0.5% across the entire curve. Total CPU time: 4h 40min 25s.

Overlay

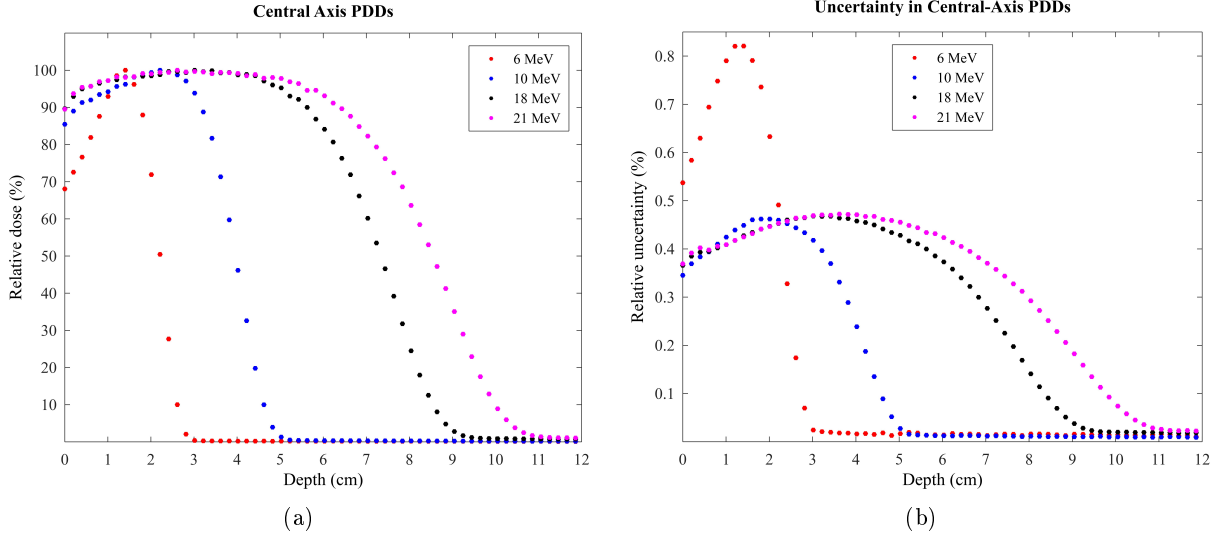


Figure 3.5: (a) Overlay of central-axis PDD curves for 6, 10, 18 and 21 MeV beams, each normalized to 100% at $Z_{D_{\max}}$. (b) Corresponding relative uncertainties, all well under the 2% clinical tolerance. Total CPU time ranged from 2h 33min 20s to 5h 8min 48s. Notice how the curves converge near half-range, indicating a common scattering equilibrium region.

3.2 Lateral dose profiles

The following section presents the lateral dose profiles along the X -axis for the four electron beam energies (see Section 1.3.5). Profiles were scored at the depth of maximum dose, $Z_{D_{\max}}$, using 64 dosels with 0.7 cm transverse resolution and normalized to $D_{\max}$. For each energy, the field size was estimated.

6 MeV

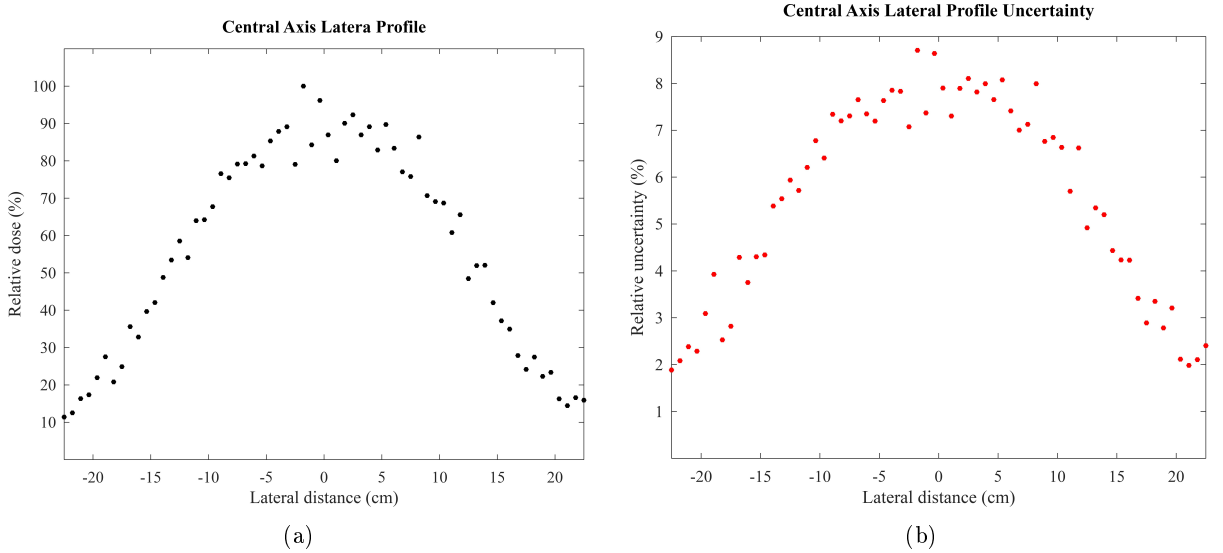


Figure 3.6: (a) Central-axis lateral dose profile for a $26.6 \times 26.6 \text{ cm}^2$ field in water irradiated by a 6 MeV electron beam, scored with 64 dosels at 0.7 cm resolution. (b) Corresponding relative uncertainty, which remains near 9% across most of the dose plateau and causes the visible noise seen in (a). Total CPU time: 3h 2min 27s.

10 MeV

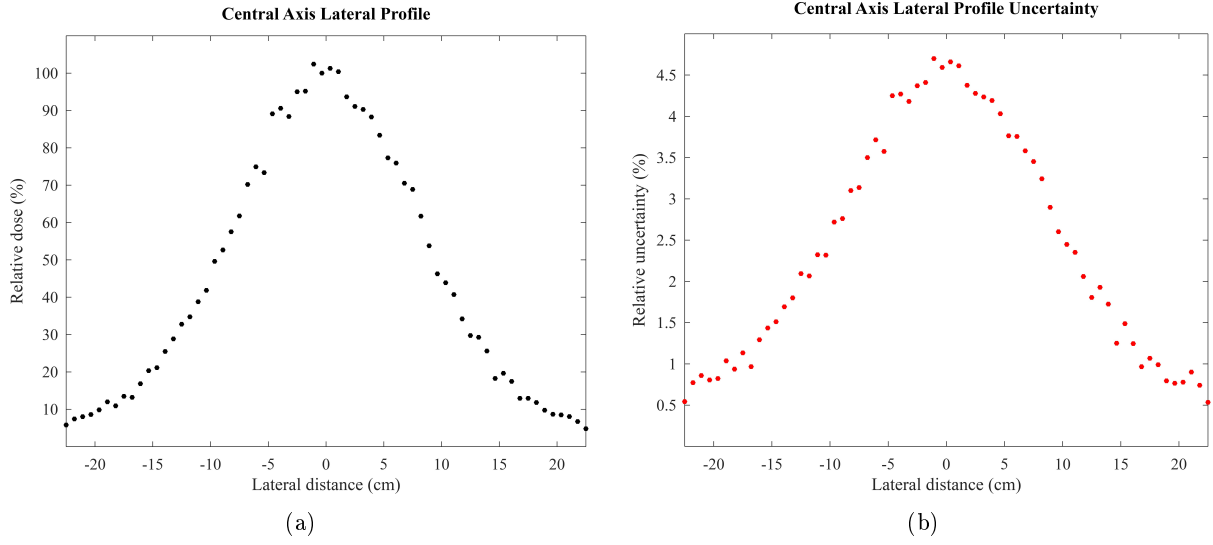


Figure 3.7: (a) Central-axis lateral dose profile for a $18.8 \times 18.8 \text{ cm}^2$ field in water irradiated with a 10 MeV beam, scored with 64 dosels at 0.7 cm resolution. (b) Relative uncertainty, which stays above 2% through most of the field. Total CPU time: 3h 45min 12s.

18 MeV

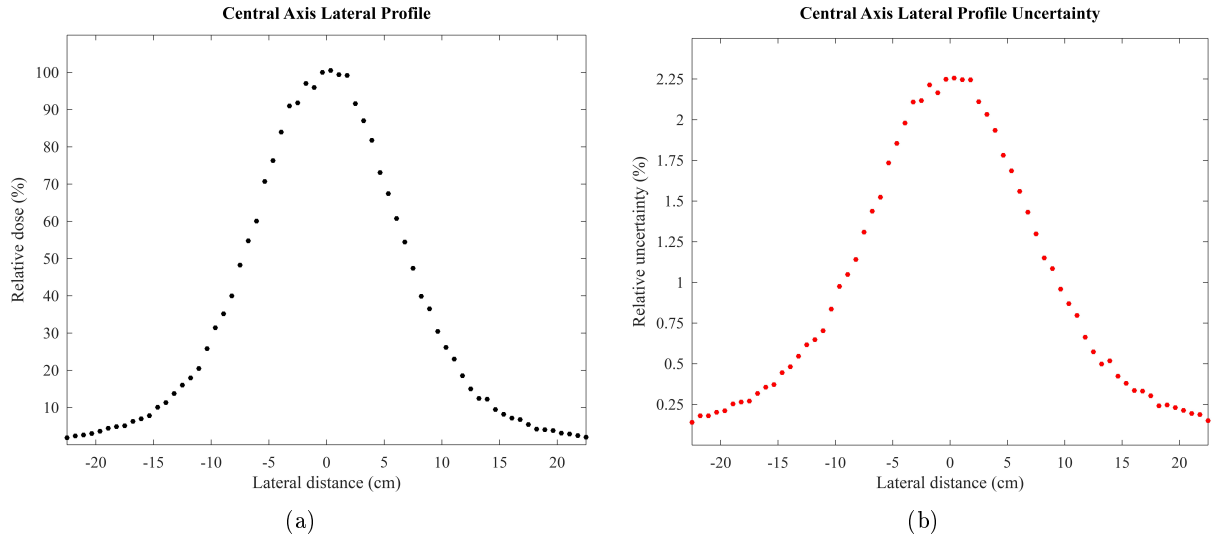


Figure 3.8: (a) Central-axis lateral dose profile for a $14.5 \times 14.5 \text{ cm}^2$ field in water irradiated with an 18 MeV beam, scored with 64 dosels at 0.7 cm resolution. (b) Relative uncertainty, kept below 2.5% throughout the entire profile. Total CPU time: 7h 12min 33s.

21 MeV

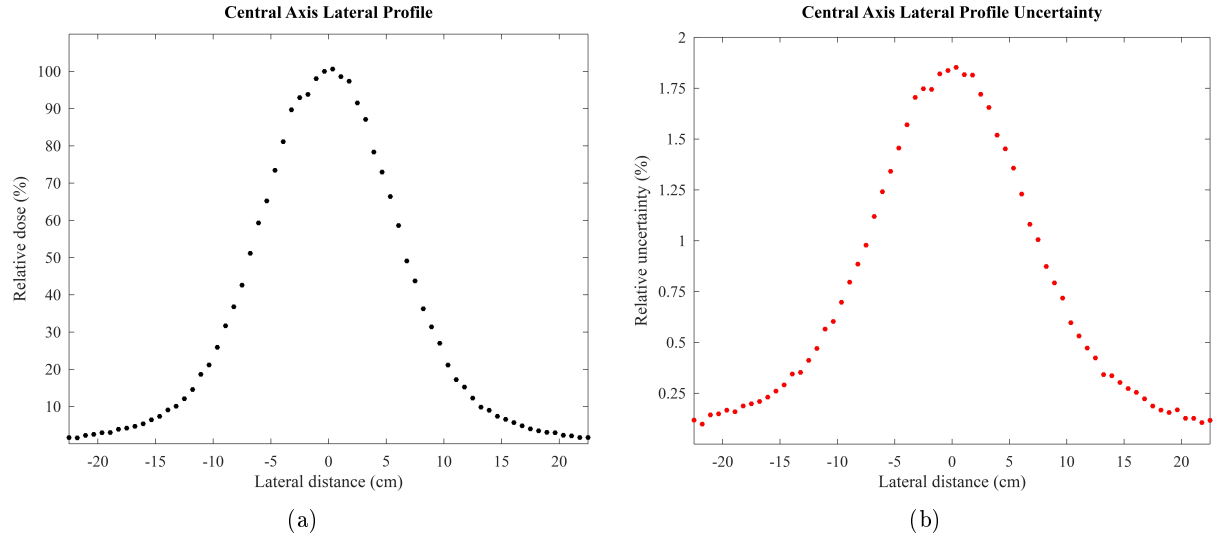


Figure 3.9: (a) Central-axis lateral dose profile for a $13.6 \times 13.6 \text{ cm}^2$ field in water irradiated with a 21 MeV beam, scored with 64 dosels at 0.7 cm resolution. (b) Relative uncertainty, maintained below the 2% threshold throughout the profile. Total CPU time: 7h 37min 05s.

Overlay

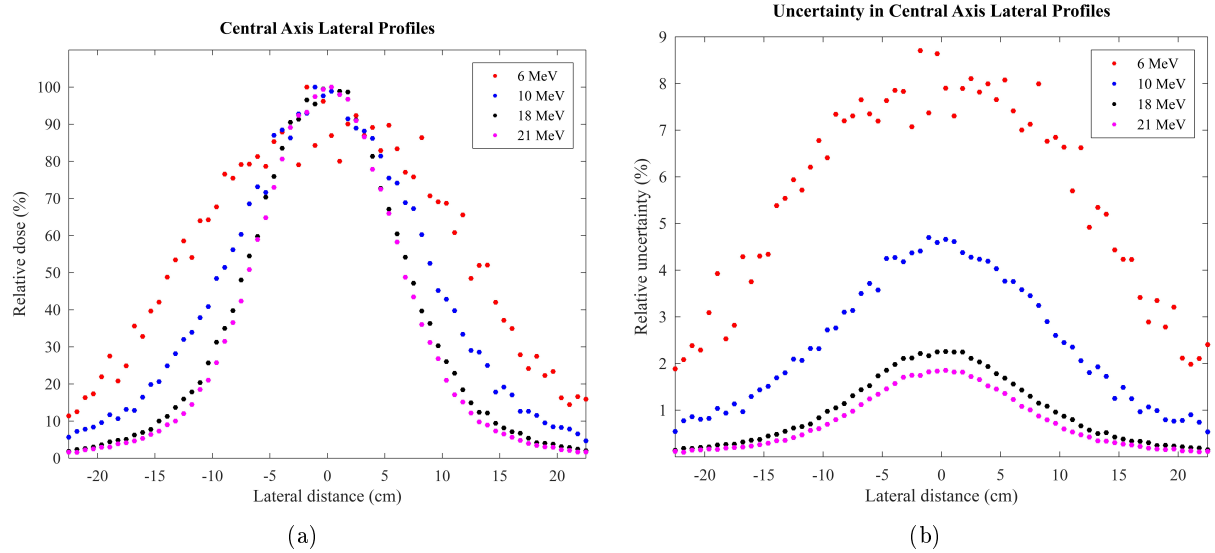


Figure 3.10: (a) Overlay of lateral dose profiles for 6, 10, 18 and 21 MeV beams, scored with 64 dosels at 0.7 cm resolution. (b) Corresponding relative uncertainties, peaking near 9% at 6 MeV and remaining below the 2% clinical tolerance at 18 and 21 MeV. Total CPU time: 7h 50min 10s. The energy-dependent narrowing of the field size is clearly visible.

3.3 Energy spectrum

The following section presents the energy spectra obtained for each of the four electron beam energies at key depths in the water phantom. For each beam energy, spectra were scored at the build-up zone, at $Z_{D_{\max}}$, at R_{90} , at R_{50} and at R_p . Simulations employed 300 energy bins to ensure fine spectral detail.

6 MeV

10 MeV

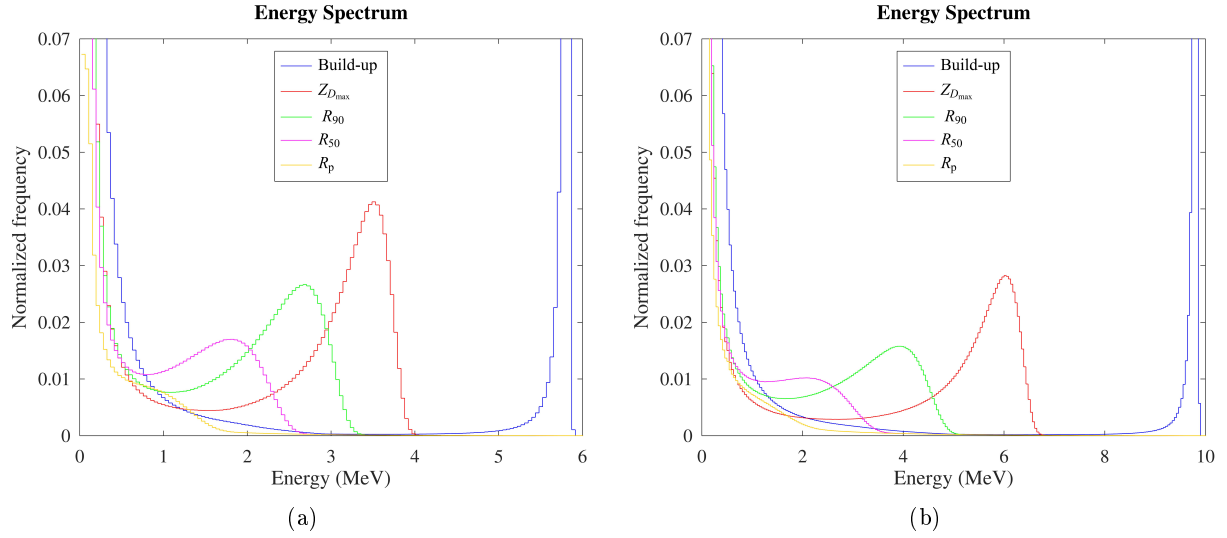


Figure 3.11: Energy spectra for monoenergetic electron beams at (a) 6 MeV and (b) 10 MeV, scored at build-up depth, $Z_{D_{\max}}$, R_{90} , R_{50} , and R_p in a water phantom. Both plots demonstrate progressive spectral broadening and a shift toward lower energies with increasing depth, reflecting cumulative in-phantom energy degradation and scattering.

18 MeV

21 MeV

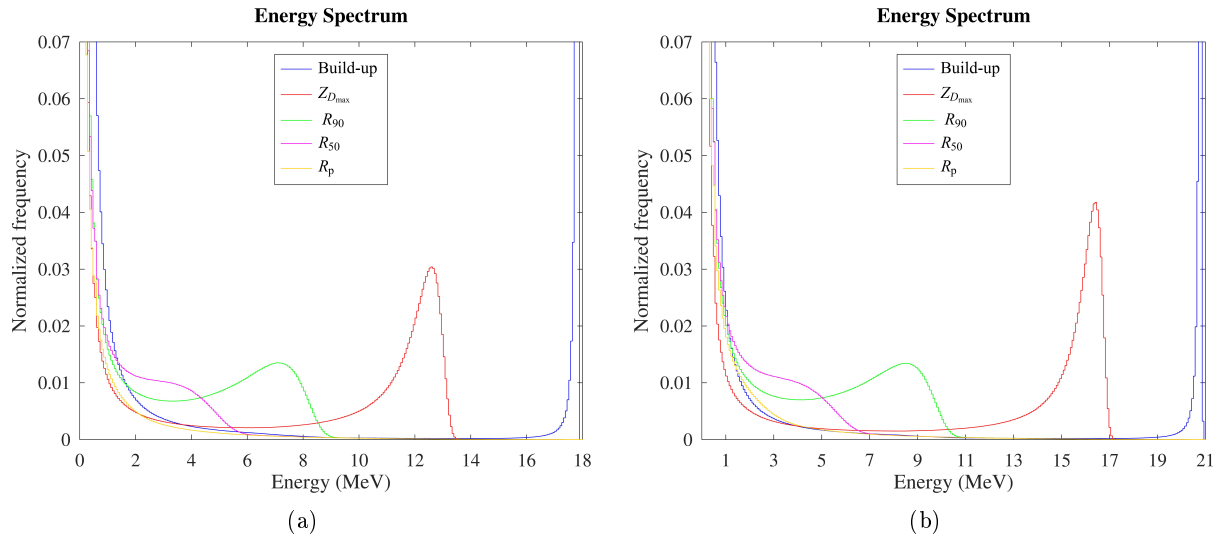


Figure 3.12: Energy spectra for monoenergetic electron beams at (a) 18 MeV and (b) 21 MeV, scored at key depths in water. The higher-energy beams retain a sharper peak deeper in the phantom, illustrating delayed energy degradation.

Overlay at the depth of maximum dose

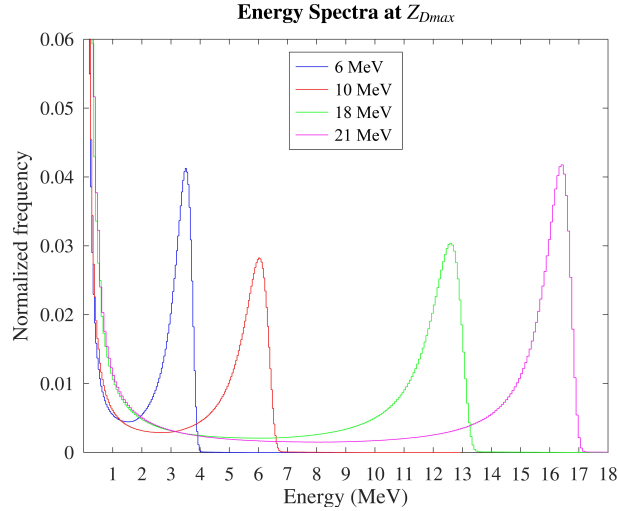


Figure 3.13: Energy spectra at $Z_{D_{\max}}$ for monoenergetic electron beams of 6, 10, 18 and 21 MeV in a water phantom.

3.4 Discussion

The simulated PDD curves (Figs. 3.1–3.4) exhibit characteristic electron beam behavior in water: initial dose build-up, pronounced $Z_{D_{\max}}$, rapid distal fall-off, and a low-intensity bremsstrahlung tail. Also, a systematic energy dependence emerges clearly: as beam energy increases from 6 to 21 MeV, $Z_{D_{\max}}$ deepens from 1.4 to 3.02 cm while the plateau extends substantially from 0.5 to 5 cm (Fig. 3.5). Energy spectra provide fundamental insight into this behavior. Analysis reveals progressive energy degradation with depth—for example, 21 MeV electrons degrade to ~ 16 MeV at $Z_{D_{\max}}$ (Fig. 3.13), while R_{50} consistently corresponds to ~ 1.8 – 2.3 MeV electrons across all beams (Figs. 3.11–3.12). This spectral evolution confirms that cumulative energy loss via inelastic collisions governs depth-dose characteristics.

Statistical uncertainties remained well below 2% at the depth of maximum dose (Figs. 3.1b–3.4b), validating the scoring methodology and demonstrating its reliability.

Table 3.1: Depth-dose parameters (Simulated/Reference [1] [3])

Energy (MeV)	R_{90} (cm)	R_{80} (cm)	R_{50} (cm)	R_p (cm)
6	1.8/1.7	1.9/1.8	2.2/2.2	2.7/2.9
10	3.2/3.1	3.4/3.3	4.0/3.9	4.7/4.8
18	5.6/5.5	6.3/5.9	7.3/7.3	9.1/9.1
21	6.4/6.6	7.2/7.5	8.6/9.0	10.1/10.5

Quantitative validation against established references (Table 3.1) shows good agreement for most parameters. Practical ranges and half ranges (R_p and R_{50}) agree within 0.4 cm across all energies. This 4 % discrepancy highlights the sensitivity of mid-range parameters.

Lateral profiles demonstrate equally significant energy dependence. Field size contracts progressively from 26.6 cm^2 at 6 MeV to 13.6 cm^2 at 21 MeV (Fig. 3.10a). Relative uncertainties range from 9% at 6 MeV to $\leq 2\%$ for ≥ 18 MeV beams in plateau regions. This uncertainty behavior correlates with broader lateral profiles at lower energies, where premature energy deposition before $Z_{D_{\max}}$ enhances off-axis scattering. To enhance profile precision—particularly at lower energies—future work should consider increased particle histories or optimized dosel dimensions, balancing spatial resolution against statistical requirements.

IV Conclusions

This work presents results from several MC simulations aimed at determining central-axis depth-dose curves, lateral profiles, energy spectra, and their associated uncertainties.

The simulation setup included a monoenergetic point source of 10^8 electrons emitted within a source-to-surface distance of 100 cm into a $45 \times 45 \times 40 \text{ cm}^3$ water phantom. Percent-depth-dose curves were sampled using 200 dosels of $0.5 \times 0.5 \times 0.2 \text{ cm}^3$; lateral profiles were scored using 64 dosels of $0.7 \times 1 \times 1 \text{ cm}^3$; and energy spectra were registered using 300 bins. Simulations were run at beam energies of 6, 10, 18, and 21 MeV, typical for radiotherapy applications.

The results matched the reference data, successfully replicating the energy-dependent trends: dose-maximum depths increased from 1.4 cm at 6 MeV to 3.02 cm at 21 MeV; plateau lengths and practical ranges deviated by ≤ 0.4 cm; and statistical uncertainties remained well below 2%. Lateral profiles exhibited energy-dependent field narrowing— $26.6 \times 26.6 \text{ cm}^2$ at 6 MeV versus $13.6 \times 13.6 \text{ cm}^2$ at 21 MeV—consistent with reduced multiple scattering at higher energies. Statistical uncertainties for the 6 MeV and 10 MeV profiles were respectively around 9% and 5% across the field, indicating the need for optimization in terms of the dose grid resolution or increasing the number of histories; for 18 and 21 MeV, the relative uncertainty remained successfully below the 2% clinical threshold.

Future studies could include more complex geometries and source models, construct phantoms more closely resembling human tissue, using materials like polystyrene or Lucite, and explore additional GATE features.

In conclusion, this work serves as an initial step into Monte Carlo calculations for radiotherapy, while also providing the essential acquaintance with the GATE toolkit for GEANT4 particle transport—a crucial milestone on my path to becoming a clinical medical physicist

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